

# DISCRETE RV HOMEWORK

TEXT: 4.1, 4.2

LAST NAME	FIRST NAME	DATE
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A company wants to evaluate its attrition rate, in other words, how long new hires stay with the company. Over the years, they have established the following probability distribution. Here  $X$  is a random variable that is equal to the number of years a new hire stays with the company before moving on, and  $P(X = x)$  is the probability that the new hire stays for  $x$  years.

$x$	0	1	2	3	4	5	6
$P(X = x)$	0.12	0.18	0.30	0.15	?	0.10	0.04

1. How likely is a new hire to stay with the company for 4 years?
2. How likely is a new hire to stay with the company for 5 or 6 years?
3. How likely is a new hire to stay with the company for an odd number of years?
4. Find  $P(X \leq 3)$
5. Find  $P(X > 2)$
6. Find  $EX$ , the expected number of years a new hire will stay with the company.
7. Find the variance of the random variable  $X$ , which is  $E(X^2) - (EX)^2$ .
8. Construct the cumulative probability distribution  $P(X \leq x)$

**9.** Denzel pays \$5 for a lottery ticket, and has to match a random three-digit number, from 000 to 999. If Denzel matches first two digits but not the last one, then he can collect \$20. If Denzel matches all three digits, then he can collect \$1000. In all other cases Denzel wins nothing.

Construct the probability mass function for the total profit in this game, which is the winnings minus the five dollars for the ticket. Find the average amount Denzel wins or loses every time he plays the game.

**10.** Consider a casino game where three fair coins are tossed, and then the player receives \$1 for each one that shows Heads, unless all three coins show Tails, and then the player has to pay the casino \$13.

Construct the probability mass function for the total profit in this game and find the expected value for the profit.

**11.** You are playing a game by drawing a card from a standard 52 card deck and replacing it. If the card is a face card, you win \$30. If it is not a face card, you pay \$2. There are 12 face cards in the standard deck. What is the expected value for the profit in this game?

**12.** You buy a lottery ticket to a lottery that costs \$10 per ticket. There are only 100 tickets available to be sold in this lottery. In this lottery there are one \$500 prize, two \$100 prizes, and four \$25 prizes. Find your expected gain or loss from buying one ticket.

Carmen and Diego are playing a game where they toss a pair of fair 6-sided dice, one blue and the other red, with sides labeled 1, 2, 3, 4, 5, and 6. If the dice show the same number, then Carmen wins \$5 from Diego. If the dice show different numbers, then Carmen loses \$1 to Diego.

13. Construct a sample space (that is, the set of outcomes) for this game.

14. Construct the probability mass function for Carmen's profit.

15. Find the average amount Carmen wins or loses every time she plays the game.

16. Who is the long-term winner in this game?

Isabel tosses a fair coin until she observes Tails, as many times as it takes. Let the random variable  $T$  count the total number of tosses. The pmf of  $T$  can be written as

$$P(T = t) = \frac{1}{2^t}$$

where  $t$  is a positive integer. So, for example,

$$P(T = 1) = 1/2^1 = 0.5$$

$$P(T = 10) = 1/2^{10} = 0.0009765625$$

This random variable has an infinite pmf, since any positive integer outcome is possible. However, the sum of the (infinitely many) pmf entries is still 1.

17. Fill out the **shown portion** of the table with pmf and cdf values.

$t$	$P(T = t)$	$P(T \leq t)$
1		
2		
3		
4		
5		
...	...	...

18. Find the probability that 3 or fewer tosses will be observed.

19. Find  $P(2 \leq T \leq 5)$

20. Find  $P(2 < T < 5)$

21. Find the probability that more than 5 tosses will be observed.

## ANSWERS

1. 0.11

3. 0.43

5. 0.4

7. 2.6219

9.	$x$	-5	15	995	$EX = -3.815 \text{ USD}$
	$P(X = x)$	0.99	0.009	0.001	

11. \$ 5.384615

13. There are 36 outcomes in total, and 6 of them show the same number on the dice:

<b>1,1</b>	1,2	1,3	1,4	1,5	1,6
2,1	<b>2,2</b>	2,3	2,4	2,5	2,6
3,1	3,2	<b>3,3</b>	3,4	3,5	3,6
4,1	4,2	4,3	<b>4,4</b>	4,5	4,6
5,1	5,2	5,3	5,4	<b>5,5</b>	5,6
6,1	6,2	6,3	6,4	6,5	<b>6,6</b>

15. \$ 0

17.

$t$	$P(T = t)$	$P(T \leq t)$
1	0.5	0.5
2	0.25	0.75
3	0.125	0.875
4	0.0625	0.9375
5	0.03125	0.96875

19. 0.46875

21. 0.03125