1. Finding Limits Using The Limit Laws

Example 1.1.

$$\lim_{x\to 1} e^{x^3-x} = e^{\lim_{x\to 1}(x^3-x)},$$

since e^t is continuous everywhere, and

$$\begin{aligned} e^{\lim_{x\to 1}(x^3-x)} &= e^0 \\ &= 1. \end{aligned}$$

Example 1.2.

$$\lim_{x\to 4^+} \frac{4-x}{|4-x|} = \lim_{x\to 4^+} \frac{4-x}{x-4},$$

since $4-x<0$ as $x\to 4^+$,

$$= -1.$$

Example 1.3.

$$\begin{aligned} \lim_{x\to 2} \frac{t^2-4}{t^3-8} &= \lim_{x\to 2} \frac{(t+2)(t-2)}{(t-2)(t^2+2t+4)} \\ &= \lim_{x\to 2} \frac{t+2}{t^2+2t+4} \\ &= \frac{4}{4+4+4} \\ &= \frac{1}{3}. \end{aligned}$$

Example 1.4.

$$\begin{aligned} \lim_{x\to\infty} \frac{1-2x^2-x^4}{5+x-3x^4} &= \lim_{x\to\infty} \frac{x^{-4}-2x^{-2}-1}{5x^{-4}+x^{-3}-3} \\ &= \frac{\lim_{x\to\infty}(x^{-4}-2x^{-2}-1)}{\lim_{x\to\infty}(5x^{-4}+x^{-3}-3)} \\ &= \frac{-1}{-3} \\ &= \frac{1}{3}. \end{aligned}$$

Example 1.5. To find $\lim_{x\to\pi^-} \ln(\sin x)$, observe that $\lim_{x\to\pi^-} \sin x = 0$. As x tends to π^-, $\sin x$ converges to 0 from above, and if we let $t = \sin(x)$, then

$$\lim_{x\to\pi^-} \ln(\sin x) = \lim_{t\to 0^+} \ln t = -\infty.$$

Note that we could not write

$$\lim_{x\to\pi^-} \ln(\sin x) = \ln(\lim_{x\to\pi^-} \sin x),$$

as $\ln(t)$ is not continuous at 0.

Example 1.6. To find $\lim_{x\to\infty} e^{x-x^2}$, let $t = x - x^2$. Since

$$\lim_{x\to\infty}(x - x^2) = \lim_{x\to\infty} x(1 - x) = -\infty,$$

we know that t tends to $-\infty$ as x tends to ∞. Hence

$$\lim_{x\to\infty} e^{x-x^2} = \lim_{t\to\infty} e^t = 0.$$